Minimality, Geometry and Simultaneity

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Abstract: I give two new uniqueness results for the standard relation of simultaneity in the context of metrical time oriented Minkowski spacetime. These results improve on the classic ones due to Malament and Hogarth, for they adopt only minimal uncontroversial assumptions. I conclude addressing whether these results should be taken to definitely refute the general epistemological thesis of conventionalism.

A long and influential tradition maintains that, within the context of Special Theory of Relativity, the determination of the relation of distant metrical simultaneity involves a distinct conventional character that is not forced either by empirical facts alone nor is uniquely definable in terms of the causal structure STR attributes to the world, namely that of Minkowski spacetime. Hence according to the conventionalist it has to be stipulated by convention. Since Malament’s infamous paper of 1977 there have been several so called Uniqueness results to counter the conventionalist argument. The problems of all these results seem to be the extreme sensitiveness to the initial assumptions made in order to prove the result itself. In this paper we deal with the minimal possible set of assumptions to derive the desired result. The plan is simple. In section 1 we set things up in order to make clear what can be taken to be the numerically minimal, and possibly theoretically weakest, set of assumptions. In section 2 we derive a new uniqueness result using such set. In section 3 we back up

1 STR from now on.
2 In what follows we take Minkowski spacetime to be an example of an affine metric space with signature (1, n-1), i.e Lorentz signature where n =4. For sake of clarity we will work with the metrical Minkowski spacetime, i.e we will be using the metrical structure rather than the causal structure. This could be somehow regarded to be question begging since for the conventionalist even the determination of the metrical structure involves a conventional element. However this temptation should be resisted. The argument is cast in the metrical language only for sake of clarity. It could be recast in terms of the causal structure alone.
every assumption of the set with a plausibility argument that, we contend, should at least persuade the conventionalist. In section 4 we prove that with a very plausible strengthened version of one of the assumptions, another, even simpler, uniqueness result is derivable. We then conclude in section 5 with a brief discussion about some possible conventionalist replies.

1. Setting things up

The alleged conventionality of distant simultaneity arises when determining relations of simultaneity for distant events within a single inertial reference frame. Let L and L′ be two parallel timelike lines in such a frame and let t₁ on L be the time of the emission of a light signal from L to L′. It is received at t₂ at L′ and immediately reflected back to arrive at L at t₃. The problem here is the determination of tₑ on L that is simultaneous with t₂. According to the conventionalist⁴ thesis there is no fact of the matter which point on the line segment L(t₁,t₃) can count as simultaneous with t₂. This is famously due to the light principle. Since the points between t₁ and t₃ are those points that are spacelike separated from t₂ it follows that the relation of topological simultaneity is just spacelike separation. Metrical simultaneity however requires singling out one point on L such that is regarded as simultaneous with t₂. According to the conventionalist argument sketched above this could be done only with a conventional stipulation. It is in fact not forced by empirical facts alone nor it is possible to determine a unique point between those that are spacelike separated from t₂ given the restrictions imposed by STR. It is easily shown that this conventional latitude amounts to the conventional choice of the ε value in the fundamental equation

ε Eq) tₑ = t₁ + ε (t₃-t₁) where 0<ε<1

The conventional choice of ε = ½ yields what is called the standard simultaneity relation.

Framing the discussion in terms of ε Eq) follows from the formulation of STR in terms of relative frames. However STR could be formulated even in the invariant language of four-dimensional Minkowskian geometry. According to this formulation STR imposes a particular kind of geometric

⁴ We will not rehearse the arguments here. There are famously two different arguments, an epistemological one and an ontological one. For classic formulation see H. Reichenbach, Philosophy of Space and Time, New York: Dover, 1957 and A. Grünbaum, Philosophical Problems of Space and Time, Dordrecht: Riedel, 1974, second edition, respectively.
structure to the world, namely that of metrical Minkowski spacetime. The conventionalist thesis could then be recaptured in this context by CoGeT (Conventionalism in Geometric Terms):

CoGeT: the relation of distant simultaneity is not uniquely definable in terms of the geometric structure of Minkowskian Spacetime.

Malament (1977) contains a theorem that seems to deem CoGeT wrong. Using the powerful resources of four-dimensional Minkowskian geometry he was able to prove that the standard simultaneity relation was an equivalence relation of simultaneity:

i) definable in terms of the geometric structure of Minkowskian spacetime $\langle M, \eta_{ab} \rangle$

via the notion of orthogonality

ii) Uniquely definable in terms of $\langle M, \eta_{ab} \rangle$

His ingenious argument hinges upon the fact that the symmetries of the geometric structure $\langle M, \eta_{ab} \rangle$ are the symmetries of an unique relation of simultaneity, namely the standard relation. However it was soon pointed out that one weakness of Malament’s result was its extreme sensibility to the initial assumptions. Spirtes pointed out that simply adding a temporal orientation to Minkowski spacetime, i.e. passing from $\langle M, \eta_{ab} \rangle$ to $\langle M, \eta_{ab}, \vec{t} \rangle$ would have spoiled Malament’s result. Moreover a full invariance under the symmetries of the spacetime structure in question was required. And some contended this was a strong theoretical requirement. In fact some of those symmetries were concerned with temporal reflections or temporal translations. But how are we to be sure that “temporal operations” of that sort do not hide an implicit assumption about distant simultaneity? A natural response will be to proceed on a case-by-case basis, but, naturally, it will be better to have a general argument. These

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5 This is not historically accurate. Malament (1977) uses only the causal structure.

6 A formal characterization is given in section 3.

7 Usually stands for a timelike vector field.

8 This is due to a technical fact. Let $L$ be a timelike line and let $o$ be a point on that line. The reflection about the orthogonal subspace to $L$ at $o$ is a symmetry of $\langle M, \eta_{ab} \rangle$ but not of $\langle M, \eta_{ab}, \vec{t} \rangle$ since it maps future timelike vectors into past timelike vectors and vice versa. Taking Spirtes’ argument into consideration CoGeT should be reformulated in CoGeT*: the relation of distant simultaneity is not uniquely definable in terms of the geometry of time oriented Minkowski spacetime. If CoGeT* is false, so is CoGeT.

9 Hogarth was able to prove a uniqueness result using a composition of two distinct symmetries. We will improve on his result using just one.

10 I personally do not find these remarks convincing. I think that they stem from a misunderstanding of the symmetry operations.
and other considerations raise the natural question of the possibility to find the minimal and weakest set of assumptions needed in order to derive the desired uniqueness result. We will do that in sections 2 and 4 with Theorem 1) and Theorem 2) respectively. We will be working with time oriented Minkowski spacetime to face the considerations raised by Spirtes (1981)\textsuperscript{11} and Sarkar/Stachel (1999)\textsuperscript{12}. Moreover we will make use of only one symmetry that is a purely spatial symmetry. Theorem 1) and 2) should be then considered improvements on some results in Hogarth (1993)\textsuperscript{13} and Malament (2007)\textsuperscript{14} since two distinct symmetries that have a temporal component are used there. We contend in section 3 that the sets of assumptions labelled (1,2,3) and (1,2',3) are the numerically minimal and theoretically weakest possible\textsuperscript{15}. As already noted by Hogarth (1993), Malament (2007) it is not possible to derive a uniqueness result in time orientable Minkowski spacetime using a single worldline. It is necessary to consider a frame\textsuperscript{16}. A frame is defined as a collection of maximal timelike parallel lines passing trough each spacetime point. It is trivially definable in geometric terms from a single worldline of the frame. Let L such a line, p a point on such a line and \(u\) an arbitrary timelike vector that spans L. Then pick an arbitrary point \(p_1\) in the underlying affine space A. \(p_1\) can be written as \(p_1 = p + pp_1\). Then the line \(L_1\) of the frame passing through \(p_1\) is defined as the set of points \(L_1\) that has such a form \((q = p + pp_1 + a_1 + au)\) where \(a\) is a real number. This discussion ensures that we are not conceding ourselves resources that are not definable in terms of the geometry of time oriented Minkowski spacetime.

2. A New Uniqueness Result. Theorem 1

**Th 1)** Let F be a frame, i.e. a collection of maximal timelike parallel lines in time oriented Minkowski spacetime \(<M,\eta_{ab}>\) and let S be an arbitrary two place relation of simultaneity defined over the points in the underlying affine space A. Suppose S satisfies


\textsuperscript{15} Although we will not be able to offer a conclusive argument for that claim. And we don’t know how to balance the merits between the two theorems.

\textsuperscript{16} See Malament (2007) for the trades off of this strategy. I will not rehearse them here.
1) $S$ is an equivalence relation, i.e. it is reflexive, symmetric and transitive and for some $L$ in $F$.

2) $\forall p \in A \exists q \in L : (p,q) \in S$

3) $S$ is $F$-invariant under spatial reflections about lines of $F$.

Formally let $\varphi: A \rightarrow A$ be an isometry of time oriented Minkowski spacetime. Then $(p,q) \in S \rightarrow (\varphi(p), \varphi(q)) \in S$

Then $S=\text{Sim}_L$ where $\text{Sim}_L$ is the standard relation defined via orthogonality, i.e $(p,q)\in S$ iff $pq \perp L$.

Proof\(^{21}\) By 2) $\exists L$ in $F$ such that 2 holds. Call it $L$. Then $\forall p \in A$ let $f(p)$ the unique point on $L$ such that $pf(p) \perp L$, and let $q$ be the intersection of the arbitrary spacelike hyperplane through $p$ with $L$, such that $(p,q) \in S$. We claim that these three conditions hold:

4) $\forall p \in A, (p, f(p)) \in \text{Sim}_L$

5) $\forall p, p' \in A (p,p') \in \text{Sim}_L$ iff $f(p) = f(p')$

6) $\forall p \in A (p, f(p)) \in S$.

(4,5,6) together imply $S = \text{Sim}_L$ by a set theoretic argument that, in the version we’ll be presenting, is essentially due to Hogarth\(^{22}\). Conditions 4 and 5 are almost immediate. 4) follows immediately from the geometric definition of simultaneity and definition of $f(p)$ and 5) follows trivially from facts about orthogonality. Condition 6 requires an argument.

We will be referring to fig. 1.

Let $L_2$ and $L_1$ be respectively the timelike lines in $F$ through $o_2 = f(p) + \frac{1}{2} f(p)p$, i.e $o_2$ is the midpoint of the line segment $L_s(f(p),p)$ and $o_1 = f(p) + \frac{1}{2} f(p)o_2$, i.e $o_1$ is the midpoint of the line segment $L_s(f(p),o_2)$.

\(^{17}\) Reflexivity is not necessary for the proof.

\(^{18}\) The full set of $F$-isometries will have to include: translations along a timelike line, arbitrary translations between different timelike lines, reflections about $L$ and reflections about timelike lines different than $L$. We use only the last one in the proofs.

\(^{19}\) We use isometry and symmetry in the same sense. Informally an isometry is a map that preserves the inner product structure, i.e the metric structure, of Minkowski spacetime. See Malament (2007) for a formal characterization.

\(^{20}\) As a conventional notation vectors are written with bold characters. Whenever we are talking about orthogonality we clearly mean Minkowskian orthogonality. Let $u$ be a vector that spans $L$, then $pq \perp L$ iff $<pq,u> = 0$, or $\eta(pq,u) = 0$.

\(^{21}\) This proof and the proof of Theorem 2 draw heavily from Malament (2008). We are assuming not more than familiarity with vector spaces, affine spaces and metric spaces.

\(^{22}\) My version should be considered a somehow cleaner version of the argument.
Consider $o_2$. By 2) there is a unique point $q$ on $L$ such that $(o_2, q) \in S$

Let $\varphi_1 : A \rightarrow A$ be a spatial reflection about $L_1$. Then we will have

7) \[ \varphi_1(o_2) = f(p). \]

By 7) and invariance in 3) we get:

8) \[ (\varphi_1(o_2), \varphi_1(q)) \in S, \text{ i.e. } (f(p), \varphi_1(q)) \in S. \]

Then let $\varphi_2 : A \rightarrow A$ be a spatial reflection about $L_2$. We will have

9) \[ \varphi_2(\varphi_1(o_2)) = \varphi_2(f(p)) = p \text{ and } \]
10) \[ \varphi_2(\varphi_1(q)) = \varphi_1(q) \]

By invariance in 3) we get
11) \((\varphi_2(\varphi_1(o_2)), \varphi_2(\varphi_1(q))) \in S\), i.e \((p, \varphi_1(q)) \in S\)

Then, by symmetry in 1), and 8)

12) \((\varphi_1(q), f(p)) \in S\)

and then by applying transitivity in 1), to 8) and 12) we get

13) \((p, f(p)) \in S\) as claimed.

This was the hard part of the problem. In the rest we prove that \(S = \text{SimL}\) follows from 4,5,6. The general structure of the argument will be rather simple. First we show that \(S \subseteq \text{SimL}\), then we show that \(\text{SimL} \subseteq S\). Hence the equality follows.

Here’s an argument for the first half of the proof. Consider two arbitrary points \(p\) and \(p'\). First assume that \((p, p') \in S\). Then

14) \((p, p') \in S\) (assumption)
15) \((p', f(p')) \in S\) (by 6)
16) \((p, f(p')) \in S\) (by transitivity in 1 applied to 14 and 15)
17) \((p, f(p)) \in S\) (by 6)
18) \(f(p') = f(p)\) (by 16, 17 and uniqueness in 2, since both \(f(p')\) and \(f(p) \in L\))
19) \((p, p') \in \text{SimL}\) (by 5).

This gives us the first half of the proof, namely that \(S \subseteq \text{SimL}\). For the second half of the proof assume that \((p, p') \in \text{SimL}\). Then

20) \((p, p') \in \text{SimL}\) (assumption)
21) \((f(p) = f(p')\) (by 5 and 20)
22) \((p, f(p)) \in S\) (=17, by 6)
23) \((p', f(p')) \in S\) (=15, by 6)
24) \((p', f(p)) \in S\) (by 21 and 23)
25) \((f(p), p') \in S\) (by symmetry in 1 applied to 24)
26) \((p, p') \in S\) (by transitivity in 1 applied to 22 and 25)

This gives us the second half of the proof, namely \(\text{SimL} \subseteq S\). Hence \(S = \text{SimL}\) and we’re done.

23 Naturally in what follows \(f(p')\) is defined with respect to \(p'\) in the same way as \(f(p)\) was defined with respect to \(p\).
Theorem 1) seems then to prove CoGeT false, at least at first sight. However the question about naturalness and minimality of assumptions 1,2,3 still lingers on. It is to this question that we now turn on.

3. Naturalness, Minimality and Assumptions

The question is, where do the assumptions 1,2,3 come from? Are they natural assumptions to make? Are they minimal? It is not the case that just labeling them as “natural” renders them so. In this section we provide what could be regarded as plausibility arguments for all of the assumptions made. The discussion will lead to another technical result that can be of some interest regarding the issue of minimality.

**Plausibility Argument for 1).** The only real concern here is transitivity. Recall the sketch of the conventionalist argument in section 1. The conventionalist holds that topological simultaneity is just spacelike separation. But the latter is not transitive. So neither is the first. The problem here, as far as we can see, is that even according to the conventionalist we are dealing here with metrical and not just topological simultaneity. Metrical simultaneity is not just spacelike separation but rather *sharing a unique spacelike hypersurface*, being this hypersurface the one that yields the standard relation of simultaneity or another. And this sharing relation is transitive. The very same point could be made in somehow classical terms. If simultaneity has to be regarded as having the same time coordinates relative to an adequate choice of an inertial frame then transitivity follows trivially. At this point the conventionalist might as well suggest an analysis of the notion of simultaneity that does away with the relation of sharing a spacelike hypersurface and that is not transitive. However we could not see any viable candidate for that.

**Plausibility Argument for 2).** We have the same kind of argument for assumption 2). When passing from topological to metrical simultaneity even the conventionalist should agree that given a line $L_n$ in $F$ there is a *unique point* $q$ in $L_n$ such that $(p,q) \in S$. The disagreement here is not in the fact that there is a unique point but rather which one is the point in question and how we can single out this point. According to the conventionalist no point that is spacelike separated from $p$ could be singled out without recurring to a conventional choice. Theorem 1) is supposed to show that it is uniquely fixed

\[24\] Grünbaum has claimed several times that one of the weakness of the original Malament’s theorem was to assume rather then prove the transitivity of simultaneity.
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by the geometric structure of Minkowski spacetime, q being the only point on L such that pq is Minkowski orthogonal to L. Note that 2) is quite a weak requirement for it is required to hold just for one timelike line L_ in F. However if our argument above is sound and is to be agreed upon even by the conventionalist we could strengthen 2) in

2’) for every L in F \( \forall p \in A \exists q \in L : (p,q) \in S \).

In the next section we will show that 2’) allows for an even easier derivation of an uniqueness result.

*Plausibility Argument for 3).* The invariance requirement in general stems out from the definability in terms of the geometric structure. If a relation R has to be defined in terms of a geometric structure it has to be invariant under its symmetries since these are those mappings that preserve that very structure. The only complain here was in the use of those symmetries that somehow incorporate suspicious *temporal operations*. But we have used only spatial reflections. Moreover we have just used one type of symmetry, so that minimality seems secured. We have however used it twice. In the next section we prove that, once 2) is replaced with 2’) 26, the very same symmetry has to be used just once, thus rendering the minimaliy argument even stronger.

4. A new Uniqueness Result. Theorem 2

*Th2).* Let F be a frame, i.e a collection of maximal timelike parallel lines in time oriented Minkowski spacetime \(<M,\eta_{ab},\uparrow>\) and let S be a two place relation defined over the points in the underlying affine space A. Suppose S satisfies

1) S is an equivalence relation, i.e it is reflexive, symmetric and transitive

2’) for every L in F \( \forall p \in A \exists q \in L : (p,q) \in S \)

3) S is invariant under rotation about lines of F \( L_1...L_n \neq L \)

Then \( S = \text{Sim}_L \) where \( \text{Sim}_L \) is the standard relation defined via orthogonality, i.e \( (p,q) \in S \text{ iff } pq \perp L \)

25 And not just for *some* L.
26 And we urge the reader to think that this is plausible enough.
Proof). As in the proof of Th1) let q be the unique point on L such that 
\((p,q) \in S\). We claim that conditions 4, 5, 6 of section 2 hold. As before 4, 5 are immediate and only 6 requires an argument. Together they imply \(S = \text{Sim}L\) by the same set theoretic argument of section 2 so we will not repeat it here and stop at proving 6). We will refer to fig 2.

By 2'\) there is a unique point o on \(L_2\) such that \((p,o) \in S\). By assumption we also have that \((p,q) \in S\). By symmetry we get

27) \((q,p) \in S\)

Then by transitivity in 1) and 2') we get

28) \((q,o) \in S\)
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Then let $\varphi: A \rightarrow A$ be a spatial reflection about $L_2$. We have

29) $\varphi(p) = f(p)$
30) $\varphi(o) = o$

By invariance

31) $(\varphi(p), \varphi(o)) \in S$, i.e $(f(p), o) \in S$

But by 2’) for every point in $A$, and hence a fortiori for $o$, there is a unique point on every line $L$ in $F$ such that is simultaneous with $o$. But $f(p)$ and $q$ both belong to the same line $L$. Hence by 2’), 27) and 31)

32) $f(p) = q$.

This yields the desired result 3), i.e $(p, f(p)) \in S$, by 27). We then proceed as in section 2 to derive $S = \text{SimL}$ and we’re done.

5. Conclusion. Epistemology to the rescue?

In this section I will consider whether the results proven in theorems Th1) and Th2) do refute conventionalism conclusively. I will argue that, though they strike a hard blow to the conventionalist core, this need not to be considered a fatal one. Yet a certain requalification of the conventionalist position is on order. Before entering the details, I want to deal briefly with a suggestion that has recently appeared in the literature, most notably in Rynasiewicz (2001)27. Rynasiewicz suggests that, despite results a la Malament, a conventional element in the determination of the relation of distant simultaneity remains in deciding to foliate the four-dimensional Minkowski spacetime in a 3+1 manifold. I have to admit that I am not sure to understand exactly what conventional element is here at stake. It can be proven with a little algebra that Minkowski spacetime is separable, i.e it is possible to foliate it in a three dimensional spacelike submanifold and a one dimensional causal28 submanifold. It is moreover possible to prove that there can not be any causal submanifold with dimension higher than one. It then follows that it is always possible to foliate Minkowski spacetime and that the 3+1 foliation is the only

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28 A submanifold is causal if every vector belonging to it is either timelike or null.
possible one. This seems to me enough to show that no conventional element is present in this particular foliation. Back to the main track. Different kind of conventionalist arguments have been advanced, in particular regarding the conventionality of metrical geometry and the conventionality of distant simultaneity. It should be noted that different kinds of arguments support different variants of conventionalism. This is particularly clear in the case of metrical geometry. The linguistic inflexion advocated in Grünbaum (1974) is different from the conventionalist variant discussed for example in Einstein (2002)\textsuperscript{29}. Discussing the conventionalist arguments for metrical geometry is beyond the scope of the paper. So I will stick to the simultaneity case. In this case, the classical conventionalist argument, to be found in Reichenbach (1957), Salmon (1969)\textsuperscript{30} and Grünbaum (1974) to name just a few, and claimed to be taken from the analysis in Einstein’s original paper on STR\textsuperscript{31}, is explicitly linked to the so called \textit{causal theory of time} (CTT). The central tenet of CTT is that temporal relations can be reduced to causal relations which are taken as primitive. In the context of CTT for every relation $R$

\[ R \text{ is Conv (for conventional) iff it is not uniquely definable in terms of the causal relations allowed by the physical theory in question.} \]

Since the theory in question is STR and the causal structure STR allows is encoded in the light cone structure of Minkowski spacetime, CoGeT seems a rather fair characterization of the main conventionalist claim. If so, I have to admit that, if the arguments in sections 2–4 are sound, then conventionalism is on the edge of being untenable. The full resources of invariant four-dimensional geometry seem to dim CoGeT false at first sight. The early conventionalist arguments simply did not take into consideration the full force of such resources. Yet I don’t believe this to be just a coincidence or the result of a certain negligence. It seems to me that this attitude can be traced back to two different and independent sources: on the one hand the thesis of conventionality of metrical geometry and on the other hand the sympathy for a relationalist ontology held by many conventionalists. A mild form of scientific realism and thus a mild form of substantivalism regarding Minkowski spacetime is required for Th1) and Th2) to have some force. Holding a relationalist ontology will surely block the main argument of the paper. Very roughly,

\textsuperscript{31} Reprinted in Einstein (2002).
there is no such a thing as Minkowski spacetime according to the relationalist. This is just a useful mathematical device that helps to simplify the complex spatiotemporal relations that obtain between physical bodies that in the end are the only entities admitted in ontology. Then no consideration based on the isometries of Minkowski spacetime could possibly force any particular spatiotemporal relation to hold or not to hold among those bodies.

The case of the conventionality of metrical geometry is less straightforward. Generally two spacetime models can have the same causal structure and yet differ in their metrical structure. Changing the metrical structure will surely result in changing isometries but not necessarily in changing the causal isomorphisms, i.e. those symmetry operations that preserve only the causal structure. And invariance under causal isomorphisms is, strictly speaking, all that is required to deliver the uniqueness results32.

But it is quite clear that both cases, the relationalist case and the case from the conventionality of metrical geometry, require substantive independent arguments. And it seems to me that providing such arguments just to assert the conventionalist claim for distant simultaneity won’t be a good general strategy. This is because I find that the case for conventionalism, if it could be made, should not rest on the causal theory of time. The variant of conventionalism supported by the causal theory of time is doomed to failure simply because a completely causal theory of time is doomed to failure. Recent works have shown that generally is not possible to recover the complete spatiotemporal structure from causal considerations alone. This is the main reason why I do believe that resting the case of conventionalism on such grounds will not yield a viable option. But, is the causal ground the only ground to assert the conventionalist claim? It is not possible to answer such a question exhaustively here, but I will however sketch an attempt arguing that this is not the case.

To see that we need to step back for a moment from the particular issue of the conventionality of distant simultaneity and look again at some general epistemological issues. The following terminology is borrowed from Reichenbach (1957). Reichenbach (1957) acknowledges two different kinds of definitions33 at work in a physical theory. The first kind is a reductive definition in which a concept is reduced to other concepts previously defined. The second kind of definition is called a coordinative definition, and, according to Reichenbach, coordinates a concept or a mathematical entity with a particular physical object or process involving physical objects.34 Let’s now return to

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32 I have used the metrical structure and so invariance under isometries just for sake of clarity.
33 The terminological choice of the word definition probably betrays a particular linguistic inflexion of the overall proposal.
34 Note that coordinative definitions in Reichenbach (1957) replace the axioms or principles of
the uniqueness results presented in Th1) and Th2) and look closely at how they are derived. Those theorems are derived using the full strength of invariant four-dimensional geometry machinery. It is this machinery that is used in the so called invariant formulation of STR. It is usually proceeded as follows. The formal theory of vector, affine and metric spaces is developed. Then Minkowski space is introduced as an example of an n-dimensional metric affine space with signature (1, 1-n). But this is still a formal geometrical theory. What is the physical significance of Minkowski space? Malament (2008) is both illuminating and clear:

“The physical significance of Minkowski spacetime structure can be explained, at least partially, in terms of a cluster of interrelated physical principles that coordinate spacetime structure with physical objects and processes”

Take for examples three of such principles which I have adapted from Malament (2008):

P1) Timelike curves represent the spacetime trajectories of massive point particles
P2) Let p and q in A be two timelike separated points. Then the length of the vector \( pq \) represents the elapsed time between p and q as recorded by an ideal free falling clock whose spacetime worldline includes p and q.
P3) Let p and q in A be two spacelike separated points. Then the length of the vector \( pq \) is the spatial distance measured by an ideal free falling rod moving in such a way that the spacetime trajectories of its ideal point sized parts are orthogonal to \( pq \).

P1)-P3) coordinate a geometrical structure, namely curves, vectors and their lengths, to physical objects, namely massive point particles, clocks and rods or processes involving them, namely measurements of distances. They seem to be the very paradigm of a coordinative definition in Reichenbach’s (1957) sense. If so the question to be asked is this. What is the epistemological status of such principles as P1-P3 that are implicitly used to derive uniqueness results like Th1) and Th2)? Do they contain a conventional element? And, if so, what is this element?

Such principles implicitly define the physical significance of Minkowski spacetime. And I have already argued that a mild form of realism is required for uniqueness results to have some force. But this physical significance of coordination in Reichenbach’s early works on relativity theory. The difference in terminology reflects a difference in the epistemological role they are supposed to play. The latter have an explicit constitutive role that the former lack.
Minkowski spaceime is often presupposed to test some of the very empirical predictions of STR, for example the dependence of a particle’s mass upon its velocity. It could be then argued\textsuperscript{35} that principles like P1-P3 have to be presupposed in order for experience to assign truth values to empirical claims of the theory. In this sense they constitute a sort of a conceptual frame of reference within which is possible to raise meaningful empirical questions. Outside such a frame many of the claims of the theory would not even have any empirical meaning. This analysis clearly links the problem of conventionalism to the recent epistemological debate over the so called dynamical relativized a priori principles\textsuperscript{36}. It seems to me that this is the most promising ground on which to rest the conventionalist claim, if any.

Let me conclude stressing a very general point. The paper and its main arguments should be read as an attempt to provide an example of how mathematical and philosophical considerations interact or should interact in the solution of a problem. A little bit of mathematical sophistication and clarity could not but help to deepen the understanding of the problem and to guide towards its solution. Yet it would be ingenuous at best to believe that a technical result by itself would provide a transparent solution that exhausts the philosophical issues that are often hidden in the core of a problem.

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\textsuperscript{35} But note that it would require a substantive argument.

\textsuperscript{36} See for example M. Friedman, \textit{Dynamics of Reason}, CSLI publications, Stanford (2001).